Finite Math - J-term 2019

Lecture Notes - 1/22/2019

Homework

- Section 5.2 - 17, 18, 19, 20, 21, 24, 33, 38, 39, 41
- Section 5.3 - 1, 2, 5, 6, 9, 17, 19, 21, 23, 25, 27

Section 5.2 - Systems of Linear Inequalities in Two Variables

Solving Systems of Linear Inequalities Graphically.

Definition 1 (Corner Point)**.** *A* corner point *of a solution region is a point in the solution region that is the intersection of two boundary lines.*

Example 1. *Solve the following system of linear inequalities graphically and find the corner points:*

$$\begin{array}{rcrcr} x & + & y & \leq & 10 \\ 5x & + & 3y & \geq & 15 \\ -2x & + & 3y & \leq & 15 \\ 2x & - & 5y & \leq & 6 \end{array}$$

Solution.

Example 2. *Solve the following system of linear inequalities graphically and find the corner points:*

$$\begin{array}{rcrcl} 5x & + & y & \geq & 20 \\ x & + & y & \geq & 12 \\ x & + & 3y & \geq & 18 \\ & & x & \geq & 0 \\ & & y & \geq & 0 \end{array}$$

Definition 2 (Bounded/Unbounded)**.** *A solution region of a system of linear inequalities is* bounded *if it can be enclosed within a circle. If it cannot be enclosed within a circle, it is* unbounded*.*

Question. *Which of the regions in the last 4 examples are bounded? Which are unbounded?*

Section 5.3 - Linear Programming in Two Dimensions: A Geometric Approach

A Simple Linear Programming Problem.

Example 3. *A food vendor at a rock concert sells hot dogs for* $4 *each and hamburgers for* $5 *each. She purchases hot dogs for* 50¢ *each and hamburgers for* $1 *each. If she has* $500 *to spend on supplies, and wants to bring at least* 100 *each of hot dogs and hamburgers, how many hot dogs and hamburgers should she buy to make the most money at the concert? (Assume she sells her entire inventory.) What is her maximum revenue?*

General Description of Linear Programming. In a *linear programming problem*, we are concerned with *optimizing* (finding the maximum and minimum values, called the *optimal values*) of a linear *objective function* z of the form

$$z = ax + by$$

where a and b are not both zero and the *decision variables* x and y are subject to *constraints* given by linear inequalities. Additionally, x and y must be nonnegative, i.e., $x \geq 0$ and $y \geq 0$.

The following theorems give us information about the solvability and solution of a linear programming problem:

Theorem 1 (Fundamental Theorem of Linear Programming)**.** *If the optimal value of the objective function in a linear programming problem exists, then that value must occur at one or more of the corner points of the feasible region.*

Theorem 2 (Existence of Optimal Solutions)**.**

(A) *If the feasible region for a linear programming problem is bounded, then both the maximum value and the minimum value of the objective function always exist.*

(B) *If the feasible region is unbounded and the coefficients of the objective function are positive, then the minimum value of the objective function exists, but the maximum value does not.*

(C) *If the feasible region is empty, then both the maximum value and the minimum value of the objective function do not exist.*

Geometric Method for Solving Linear Programming Problems.

Procedure (Geometric Method for Solving a Linear Programming Problem with Two Decision Variables)**.**

(1) *Graph the feasible region. Then, if an optimal solution exists according to Theorem 2, find the coordinates of each corner point.*

(2) *Construct a* corner point table *listing the value of the objective function at each corner point.*

(3) *Determine the optimal solution(s) from the table in Step 2 (smallest=minimum, largest=maximum).*

(4) *For an applied problem, interpret the optimal solution(s) in terms of the original problem.*

Example 4. *Maximize and minimize* $z = 3x + y$ *subject to the inequalities*

$$\begin{aligned} 2x + y &\leq 20 \\ 10x + y &\geq 36 \\ 2x + 5y &\geq 36 \\ x, y &\geq 0 \end{aligned}$$

Solution.

Example 5. *Maximize and minimize* $z = 2x + 3y$ *subject to*

$$\begin{aligned} 2x + y &\geq 10 \\ x + 2y &\geq 8 \\ x, y &\geq 0 \end{aligned}$$

Solution. *Minimum of* $z = 14$ *at* $(4, 2)$. *No maximum.*

Example 6. *Maximize and minimize* $P = 30x + 10y$ *subject to*

$$\begin{aligned} 2x + 2y &\geq 4 \\ 6x + 4y &\leq 36 \\ 2x + y &\leq 10 \\ x, y &\geq 0 \end{aligned}$$

Solution. *Minimum of* $P = 20$ *at* $(0, 2)$. *Maximum of* $P = 150$ *at* $(5, 0)$.

Example 7. *Maximize and minimize* $P = 3x + 5y$ *subject to*

$$\begin{aligned} x + 2y &\leq 6 \\ x + y &\leq 4 \\ 2x + 3y &\geq 12 \\ x, y &\geq 0 \end{aligned}$$

Solution. *No optimal solutions.*